"Theory of Elastic Stability"
Book By Timoshenko & Gere

Eq (8-25)

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left[ q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y}\right] \quad \text{.........} (*)$$

$q$ → Lateral Load

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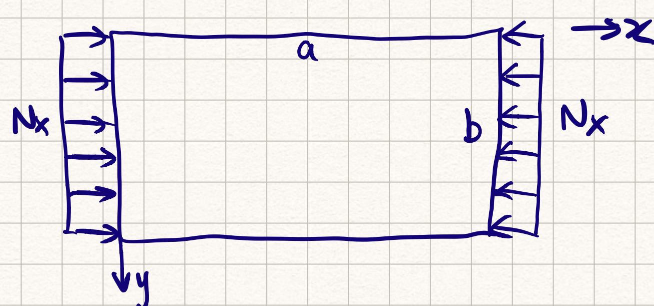
## Special Cases :

(1) SS all edges, Rectangular Plate
Uniformly compressed, in one direction




$N_x$ : compressive force per unit length

"The corresponding critical value of the compressive force can be found in this case by integrating Eq (*)"

— For simply-supported case, the deflection surface of buckled plate can be represented as :

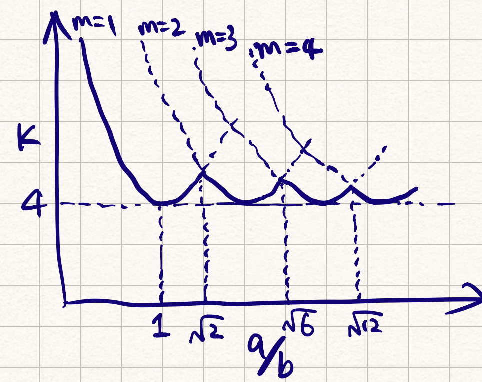
$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b}$$

⇒ compressive force :

$$N_x = \frac{\pi^2 a^2 D}{m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2$$

$$N_x \xrightarrow{n=1} (N_x)_{min} = (N_x)_{cr}$$

so $(N_x)_{cr} = \frac{\pi^2 D}{a^2}\left(m + \frac{1}{m}\frac{a^2}{b^2}\right)^2 = \frac{\pi^2 D}{b^2}\left(\frac{mb}{a} + \frac{a}{mb}\right)^2$




- $k = \left(\frac{b}{a} + \frac{a}{b}\right)^2 \Leftarrow m=1$
- $k = \left(\frac{2b}{a} + \frac{a}{2b}\right)^2 \Leftarrow m=2$
- $k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$

$(N_x)_{cr}$ is per unit length

① If $a<b$ : & $m=1$

$\left[(N_x)_{cr}\right]_{min} \Leftarrow m=1$

$\left.\begin{matrix} m=1 \\ n=1 \end{matrix}\right\} \Rightarrow w = a_{11}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$

$\Rightarrow (N_x)_{cr} = \frac{\pi^2 D}{b^2}\left(\frac{b}{a} + \frac{a}{b}\right)^2$

② If $m=2$, $\sqrt{2} < \frac{a}{b} < \sqrt{6}$

$w = a_{21}\cdot\sin\frac{2\pi x}{a}\cdot\sin\frac{\pi y}{b}$

$(N_x)_{cr} = \frac{\pi^2 D}{b^2}\left(\frac{2b}{a} + \frac{a}{2b}\right)^2$

- When $\frac{a}{b} = \sqrt{m(m+1)}$, m half-wave = (m+1) half-wave
- Critical compressive stress

$\sigma_{cr} = \frac{(N_x)_{cr}}{h} = k\frac{\pi^2 E}{12(1-\nu^2)}\frac{h^2}{b^2}$

($h$ ↙ thickness)

- Summary : In this case, $a/b$ known
  - n always equals to 1 $\Rightarrow (N_x)_{cr} = \frac{\pi^2 D}{b^2}\left(\frac{mb}{a} + \frac{a}{mb}\right)^2$
  - $(N_x)_{cr} = F(m)$ = a function of m
  - Question becomes finding m value to get minimum $(N_x)_{cr}$
  - for $m = 1 : 10^{100}$

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for m = 1 : 10^100
    S1 = sqrt(m (m+1))
    S2 = sqrt((m+1)(m+2))
    if S1 <= a/b <= S2
        break;
    end
end
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$\Rightarrow m \Rightarrow (N_x)_{cr}$